**Lab 3: Graphing Processes Modeled by Exponentials**

**Save this document to your own computer. As you work through the lab, fill in tables and copy and paste results from MATLAB as needed.**

Many physical processes can be modeled using the exponential function. Some examples include charging or discharging a capacitor, population growth or decay, heating and cooling, radioactive decay, and chemical reaction rates.

1. **Decomposition of Hydrogen Peroxide**

First order chemical reactions can be modeled using exponential functions. Hydrogen Peroxide (H2O2) decomposes as a 1st order reaction into water and oxygen gas:

The concentration of hydrogen peroxide decreases exponentially according to the following equation:

C(t) = Concentration at time t (M or mols/L)

C0 = Initial Concentration (M)

k = Reaction rate (s-1 )

The reaction rate, k, depends on the temperature according to Arrhenius’ Equation (also exponential):

A = Frequency Factor (s-1 )

Ea = Activation Energy (J/mol)

R = Ideal Gas Constant = 8.314 (J/(mol\*K))

T = Absolute Temperature (K)

1. Assume Ea = 7.5\*104 J/mol and A = 1011 s-1. Complete the following table to calculate the reaction rate for each of the given temperatures. When completed, check your values with your T.A. The value of k should be calculated to five places behind the decimal point – use the command format long in MATLAB. Remember e-3t is exp(-3\*t) in MATLAB.

|  |  |  |  |
| --- | --- | --- | --- |
| **Temperature (oF)** | **Temperature (oC)** | **Absolute Temperature (K)** | **Reaction Rate, k (s-1)** |
| 50 |  |  |  |
| 77 |  |  |  |
| 104 |  |  |  |

1. Assume the initial concentration, Co of the hydrogen peroxide is 1 M. In MATLAB, on the same graph, plot the concentration, C, of the hydrogen peroxide vs. time for all three temperatures. The concentration should be on the y-axis and time should be on the x-axis.

Hints:

* Remember in MATLAB, e-3t is entered as exp(-3\*t)
* You will need to think about a good time range for your plot. A good rule of thumb is to remember that e-5 is reasonably close to zero.

1. Add the following to your plot then copy and paste your plot into this document.

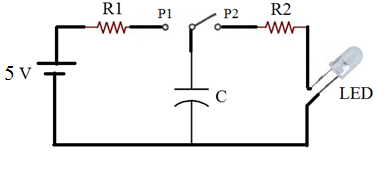
* Label the x-axis as time (s)
* Label the y-axis as Concentration (M)
* Add a title: Decomposition of Hydrogen Peroxide
* Add a legend that shows the three temperatures in oF

1. Use your plot and the Data Cursor Tool to complete the following table.

|  |  |  |  |
| --- | --- | --- | --- |
| **Temperature (oF)** | **50oF** | **77oF** | **104oF** |
| Time required for concentration to drop to ½ of the initial concentration |  |  |  |
| Time required for concentration to drop to ¼ of the initial concentration |  |  |  |
| Time required for concentration to drop to 10% of the initial concentration |  |  |  |

1. **RC Circuits**

The figure below shows a simple RC (Resistor Capacitor) circuit. When the switch is in position P1, the capacitor will charge at an exponential rate to a voltage equal to the battery voltage (5 V in this circuit). When the switch is in position P2, the capacitor will discharge at an exponential rate to 0 V. While the capacitor is discharging, it will act as a temporary battery to light up the LED (light emitting diode). The time it takes for the capacitor to charge or discharge depends on the size of the resistors and the capacitor, specifically on the product, RC, referred to as a time constant.



***Capacitor Charging***

The equation that describes how a capacitor charges over time in an RC circuit is:

Vc(t) = Capacitor voltage at time t (V)

E = Voltage of the battery or source (V)

τ is the RC time constant (s)

The time constant, τ, is simply the product of the resistance and the capacitance:

τ = RC time constant (s)

R = Resistance in ohms (Ω)

C = Capacitance in farads (F)

1. Calculate the time constant for each of the resistances shown in the table below.

Note: To calculate τ in seconds, capacitance must be in farads (F) and resistance must be in ohms (Ω). 1 µF = 10-6 F and 1 kΩ = 103 Ω.

Hint: In MATLAB, you can enter the value 4.7\*10-6 as 4.7\*10^-6 or as 4.7e-6

|  |  |  |
| --- | --- | --- |
| **Resistance** | **Capacitance** | **τ (s)** |
| 1 kΩ | 4.7µF |  |
| 2 kΩ | 4.7µF |  |
| 10 kΩ | 4.7µF |  |

1. Assume the battery voltage is 5V. On the same graph in MATLAB, plot the capacitor voltage for all three resistance values. Determine a good time range.
2. Add the following to your plot then copy and paste your plot into this document.

* Label the x-axis as time (s)
* Label the y-axis as Capacitor Voltage (V)
* Add a title: Capacitor Charging in RC Circuit
* Add a legend that shows the three resistance values

1. Using the data cursor tool, determine how long it takes the capacitor voltage to reach 63.2% and 98.2% of its final value (final value is battery voltage). Enter results in the table below:

|  |  |  |  |
| --- | --- | --- | --- |
| **Resistance** | **Capacitance** | **Time to Reach**  **63.2% of 5V** | **Time to Reach**  **98.2% of 5V** |
| 1 kΩ | 4.7µF |  |  |
| 2 kΩ | 4.7µF |  |  |
| 10 kΩ | 4.7µF |  |  |

**Questions:**

1. How is the time required to reach 63.2% of the final voltage related to the time constant, τ?
2. Approximately how many time constants does it take the capacitor to reach 98.2% of its final voltage?